

Broadcasting of continuous variable entanglement

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We present a scheme for broadcasting of continuous variable entanglement. We show how an initial two-mode squeezed state of the electromagnetic field shared by two distant parties can be broadcasted into two nonlocal bipartite entangled states. Our protocol uses a local linear amplifier and a beam splitter at each end. We compute the fidelity of the output entangled states and show that the broadcasting can be implemented for a variety of input squeezed states and amplifier phases.

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Quantum entanglement is now recognized as a powerful resource in communication and computation protocols[1]. The first nontrivial consequence of entanglement on quantum ontology was noticed many years ago within the context of continuous variable systems[2]. In recent times there has been a rapid development of the theory of entanglement pertaining to infinite dimensional Hilbert spaces[3]. Many well-known results of discrete variable systems relating to classification and manipulation of entanglement take novel forms for the case of continuous variables[4]. There still remains a lot to be understood in the information theory for continuous variables which has potentially vast practical ramifications.

An interesting issue is that of broadcasting of quantum entanglement, viz., whether the entanglement shared by a single pair can be transmitted to two less entangled pairs by local operations at both ends. Unlike classical correlations, quantum entanglement cannot always be broadcasted, as has been proved for general mixed states in finite dimensions[5]. Since broadcasting involves copying of local information, and the exact cloning of an unknown quantum state is impossible, the no-cloning theorem[6] and its consequences imply limitations on this procedure. For the case of pure states in finite dimensions, implementation of broadcasting imposes restrictions on the initial state[7] and conditions on the fidelity of the cloning process[8]. No scheme has yet been proposed, however, for the broadcasting of continuous variable entanglement.

The cloning of continuous quantum variables has nonetheless, been studied by several authors. Various schemes for duplication of coherent states with optimal fidelity and economical means have been suggested[9]. Operations of cloning machines with networks of linear amplifiers and beam splitters have been proposed[10, 11]. It is thus relevant to investigate whether such ideas of copying local information can be elaborated for broadcasting entangled states of continuous variables. To this end we extend the procedure of cloning of a single-mode squeezed state of the electromagnetic field proposed by Braunstein et al.[10] to the case of a bipartite entangled two-mode squeezed state. By applying a linear amplifier and a beam splitter available locally with each party, we

show using the covariance matrix approach[4] how the initial entangled state can be broadcasted into two non-local and bipartite entangled states.

Before describing our scheme for broadcasting in detail, it is instructive to review briefly how the copying of local continuous variable information[10] can be understood in terms of the covariance matrix approach. For this purpose, we begin with a single-mode squeezed vacuum state of the electromagnetic field represented by the squeezing transformation operator

$$S_i(r) = \begin{pmatrix} e^r & 0 \\ 0 & e^{-r} \end{pmatrix} \quad (1)$$

acting on the vacuum mode, and r is the squeezing parameter ($r > 0$). The covariance matrix (CM) corresponding to the single-mode (say, i) squeezed vacuum state is given by

$$\sigma_i(r) = S_i(r)S_i^T(r) = \begin{pmatrix} e^{2r} & 0 \\ 0 & e^{-2r} \end{pmatrix} \quad (2)$$

The cloning of this state proceeds as follows. First, a linear amplifier[4] mediates the interaction between the mode i and an ancilla (say, a) prepared in the vacuum state, which is represented by the linear transformation

$$\mathcal{A}_{ia}(r, \phi) = [S_{ia}(r, \phi)] \cdot [S_i \oplus I_a] \quad (3)$$

with ϕ being the phase of the amplifier. After this interaction the squeezed state mode together with the ancilla mode and another blank state mode (say b) are incident on a three mode 50 : 50 beam splitter B_{iab} which we define through a symplectic transformation as

$$B_{iab} = \begin{pmatrix} \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & \sqrt{1/2} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & 0 & 0 & -\sqrt{1/2} \end{pmatrix} \quad (4)$$

The above beam splitter is defined in such a way that it does not affect the ancilla mode. The total cloning operation is thus represented by the transformation

$$T_{iab}(r, \phi) = [B_{iab}] \cdot [\mathcal{A}_{ia} \oplus I_b] \quad (5)$$

with the corresponding CM given by

$$\sigma_{iab} = T_{iab}(r, \phi) T_{iab}^\dagger(r, \phi) \quad (6)$$

This procedure leads to symmetric cloning resulting in the two cloned output modes i and b .

The fidelity of the two clones can be evaluated through the relation[12]

$$F = \frac{1}{\sqrt{\text{Det}[\sigma_{in} + \sigma_{out}] + \delta - \sqrt{\delta}}} \quad (7)$$

where σ_{in} is given by Eq.(2), and σ_{out} is obtained by tracing out the ancilla mode from the CM in Eq.(6), i.e.,

$$\sigma_{out} = \begin{pmatrix} P & 0 \\ 0 & M \end{pmatrix} \quad (8)$$

with

$$P = \frac{e^{2r}(c - hs)^2 + k^2 s^2 + 1}{2}, \quad M = \frac{e^{-2r}(c + hs)^2 + k^2 s^2 + 1}{2} \quad (9)$$

$$\delta = 4(\text{Det}[\sigma_{in}] - 1/4)(\text{Det}[\sigma_{out}] - 1/4)$$

where

$$c = \text{Cosh}(2r), \quad s = \text{Sinh}(2r), \quad h = \text{Cos}(2\phi), \quad k = \text{Sin}(2\phi) \quad (10)$$

The fidelity for the above phase sensitive cloning machine is thus given by

$$F = \frac{1}{\sqrt{(P + e^{2r})(M + e^{-2r}) + 3(PM - 1/4) - \sqrt{3(PM - 1/4)}}} \quad (11)$$

If the phase of the amplifier is set to $\phi = 0$, the fidelity becomes

$$F = \frac{2}{\sqrt{8c^2 + 12c + 5 - \sqrt{3 + 6c}}} \quad (12)$$

Since the fidelity of the clones depend on the input state, the cloning is said to be state-dependent. From Eq.(12) it follows that as $r \rightarrow \infty$, $F \rightarrow 0$, and as $r \rightarrow 0$, $F \rightarrow 1$.

Let us now consider a continuous variable entangled state (an entangled state of the electromagnetic field) which is shared by two parties located far apart at sites I and J, respectively, and is represented by the generic two-mode (i and j) squeezed state with one mode at each end. The two-mode squeezed vacuum state is obtained by applying the transformation[4]

$$T_{ij}(r) = B_{ij}(1/2) \cdot (S_i(r) \oplus S_j(r)) \quad (13)$$

on two uncorrelated squeezed vacuum modes ($S_i(r)$ and $S_j(r)$) given by Eq.(1). $B_{ij}(1/2)$ denotes a balanced 50 : 50 beam splitter with the matrix form

$$B_{ij}(1/2) = \begin{pmatrix} \sqrt{1/2} & 0 & \sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & \sqrt{1/2} \\ \sqrt{1/2} & 0 & -\sqrt{1/2} & 0 \\ 0 & \sqrt{1/2} & 0 & -\sqrt{1/2} \end{pmatrix} \quad (14)$$

The CM corresponding to the two-mode squeezed state is given by

$$\sigma_{ij}(r) = T_{ij}(r) T_{ij}^\dagger(r) = \begin{pmatrix} c & 0 & s & 0 \\ 0 & c & 0 & -s \\ s & 0 & c & 0 \\ 0 & -s & 0 & c \end{pmatrix} \quad (15)$$

The two-mode squeezed vacuum state is the quantum optical representative for bipartite continuous variable entanglement. In the Heisenberg picture the quadrature operators of the two-mode squeezed state are given by

$$\begin{aligned} \hat{x}_i &= \frac{e^r \hat{x}_i^{(0)} + e^{-r} \hat{x}_j^{(0)}}{\sqrt{2}}, & \hat{p}_i &= \frac{e^{-r} \hat{p}_i^{(0)} + e^r \hat{p}_j^{(0)}}{\sqrt{2}} \\ \hat{x}_j &= \frac{e^r \hat{x}_i^{(0)} - e^{-r} \hat{x}_j^{(0)}}{\sqrt{2}}, & \hat{p}_j &= \frac{e^{-r} \hat{p}_i^{(0)} - e^r \hat{p}_j^{(0)}}{\sqrt{2}} \end{aligned} \quad (16)$$

where the superscript (0) denotes the initial vacuum modes, the operators \hat{x} and \hat{p} represent the electric quadrature amplitudes (the real and imaginary parts, respectively, of the mode's annihilation operator).

Now, for broadcasting of the above two-mode squeezed state we apply local cloning machines on the two individual modes of the entangled bipartite state, located at the sites I and J, respectively. Our scheme for broadcasting proceeds as follows. The local cloner acting on the mode at site I copies the information available locally on to two modes (i and b). Similarly, the cloner acting on the mode at site J copies information on to two modes (j and b'). Note that since the two modes on which the cloners act at sites I and J, respectively, are the constituents of an initially entangled bipartite state, the forms of the output local clones will be different in general, from the outputs of the cloning for a single-mode squeezed state given by Eq.(8). More importantly, the properties of entanglement between the output states are now dependent on the initial entangled state to be broadcasted.

The criteria for successful broadcasting[7, 8] of the entangled two-mode squeezed state (15) can be elaborated as follows. The initial entangled state is broadcasted if the local pairs of output modes (i and b at site I) and (j and b' at site J) are separable, and also if simultaneously, both the non-local pairs of output modes (i and b' on one hand), and (j and b on the other) are entangled. Our aim is to verify whether the above conditions are satisfied for the output states when the two cloners act at their respective sites. To this end, we formulate this bi-local cloning procedure using the CM approach[4].

The broadcasting operation is implemented through an ancilla mode, a linear amplifier and a beam splitter located at both sites. Thus, after introducing the ancillas a and a' (at the ends I and J, respectively), the CM of the joint two-mode squeezed state with the ancillas takes

the form

$$\sigma_{iaja'}(r) = \begin{pmatrix} c & 0 & 0 & 0 & s & 0 & 0 & 0 \\ 0 & c & 0 & 0 & 0 & -s & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ s & 0 & 0 & 0 & c & 0 & 0 & 0 \\ 0 & -s & 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (17)$$

Next, both parties apply linear amplifiers on their respective modes and ancillas. The local amplifiers can be jointly represented as

$$\mathcal{A}(r, \phi) = \begin{pmatrix} \mathcal{A}_1 & 0 \\ 0 & \mathcal{A}_2 \end{pmatrix} \quad (18)$$

where the \mathcal{A}_i are given by[4]

$$\mathcal{A}_i = \begin{pmatrix} c - hs & 0 & ks & 0 \\ 0 & c + hs & 0 & -ks \\ ks & 0 & c + hs & 0 \\ 0 & -ks & 0 & c - hs \end{pmatrix} \quad (19)$$

for $i = 1, 2$. After interaction with the local amplifiers, the CM of the two modes with their ancillas is transformed to

$$\sigma'_{iaja'}(r, \phi) = \mathcal{A}^T \sigma_{iaja'}(r, \phi) \mathcal{A}. \quad (20)$$

Thereafter, both parties introduce their respective blank modes (b and b') on which the information of the original modes is to be copied. Both parties now have three local modes each, i.e., original, ancilla, and blank mode, which fall on a 50 : 50 beam splitter defined through a symplectic transformation in Eq.(4), at each end. The two local beam splitters can be represented jointly by

$$B = \begin{pmatrix} B_{iab} & 0 \\ 0 & B_{ja'b'} \end{pmatrix} \quad (21)$$

and the resultant CM at the end of the cloning processes at both the ends is given by

$$\sigma''_{iabja'b'} = B^T \sigma'_{iabja'b'} B \quad (22)$$

Using the above CM we can now check if the criteria for successful broadcasting are satisfied. In order to verify the entanglement of the non-local pairs of modes shared by the two sides, we obtain the reduced CMs corresponding to these modes, which (after tracing out the remaining modes from $\sigma''_{iabja'b'}$) are given by

$$\begin{aligned} \sigma_{ib'}^{\text{non-local}}(r, \phi) &= \sigma_{jb}^{\text{non-local}}(r, \phi) \\ &= \begin{pmatrix} \frac{G+1}{2} & 0 & \frac{E}{2} & 0 \\ 0 & \frac{H+1}{2} & 0 & \frac{-E}{2} \\ \frac{E}{2} & 0 & \frac{G+1}{2} & 0 \\ 0 & \frac{-E}{2} & 0 & \frac{H+1}{2} \end{pmatrix} \end{aligned} \quad (23)$$

where

$$\begin{aligned} E &= s(c - hs)^2, \quad G = (c - hs)^2 c + ks^2, \\ H &= (c + hs)^2 c + ks^2 \end{aligned} \quad (24)$$

The condition for separability of the modes can be obtained from the generalization of the positivity of partial transposition (PPT) criterion for continuous variable systems[13]. For a two-mode state represented by the CM (23), the necessary and sufficient condition for the separability of these modes[4] reduces to the relation $-E^2/4 > 0$ being satisfied. Since the above inequality is always violated, it follows that the non-local output modes are always entangled.

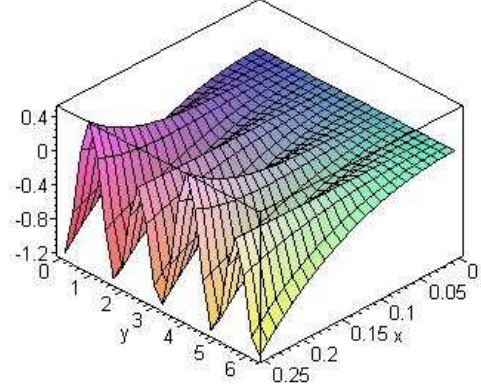


FIG. 1: The separability parameter R is plotted versus the squeezing r (x -axis), and the amplifier phase ϕ (y -axis). The condition for separability of the local modes ($R > 0$) is satisfied for various combinations of r and ϕ .

Our remaining task for implementing broadcasting is to find the conditions under which the local output modes turn out to be separable. After tracing out the ancilla mode on site I and also all the modes on the site J from $\sigma''_{iabja'b'}$, the reduced CM representing the system of the two clones on site I (which is also equal to the corresponding reduced CM on the site J) is given by

$$\begin{aligned} \sigma_{ib}^{\text{local}}(r, \phi) &= \sigma_{jb'}^{\text{local}}(r, \phi) \\ &= \begin{pmatrix} \frac{G+1}{2} & 0 & \frac{G-1}{2} & 0 \\ 0 & \frac{H+1}{2} & 0 & \frac{H-1}{2} \\ \frac{G-1}{2} & 0 & \frac{G+1}{2} & 0 \\ 0 & \frac{H-1}{2} & 0 & \frac{H+1}{2} \end{pmatrix} \end{aligned} \quad (25)$$

The PPT condition[13] for the local output modes represented by the CM (25) is satisfied[4] if the relation

$$R = [(c - hs)^2 c + (ks^2 - 1)][(c + hs^2)c + (ks^2 - 1)] > 0 \quad (26)$$

holds. It follows from Eq.(26) that whenever $ks^2 > 1$, the output local clone modes are separable. We display

in Fig.1, the three-dimensional plot of the variable R as a function of the squeezing parameter r and the amplifier phase ϕ . One sees that this criterion for broadcasting can be satisfied for several values of the above two parameters. The pattern of R versus ϕ continues for large values of squeezing too. For example, if the phase of the amplifiers is set to $\phi = \pi/4$, then broadcasting is possible for all values of the squeezing parameter r , i.e., for all continuous variable bipartite entangled states that are represented by the two-mode squeezed state (15).

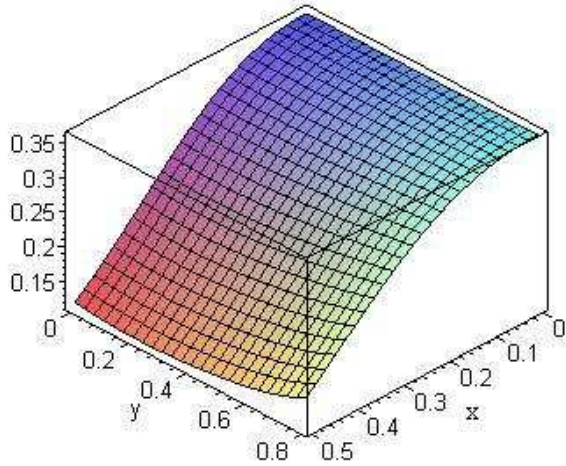


FIG. 2: The fidelity of broadcasting F_B is plotted versus the squeezing r (x -axis), and the amplifier phase ϕ (y -axis).

We have shown that broadcasting is possible in our scheme for various combinations of input states and amplifier phases. Finally, one would like to investigate the efficiency of broadcasting through this scheme. For this purpose we compute the fidelity of the broadcasted states, i.e., the entangled non-local states (23). The fidelity of broadcasting, F_B is obtained through Eq.(7), in which we substitute the expressions for σ_{in} and σ_{out} from Eqs.(15) and (23) respectively. In Fig.2 we display F_B as a function of the squeezing parameter r and the amplifier phase ϕ . One sees that this scheme of ours yields a phase- and state-dependent broadcasting fidelity. From the obtained expression for F_B it is possible to see that for $r \rightarrow \infty$, $F_B \rightarrow 0$, and for $r \rightarrow 0$, $F_B \rightarrow 0.36$.

To summarize, in this paper we have presented a scheme for broadcasting of continuous variable entanglement for the first time. We consider an initial two-mode squeezed state which is a generic bipartite entangled state of the electromagnetic field. This state is shared by two distant parties who individually apply local cloning machines on their respective modes. The cloning process[10] at each end requires an ancilla state, a linear amplifier and a beam splitter, yielding two symmetric cloned modes each. The initial state is broadcasted into a pair of bipartite entangled states that is finally shared by the two distant parties. We show using the covariance matrix

formalism[4] when the output states with the two distant parties satisfy the criteria required for successful broadcasting. We also compute the fidelity of broadcasting using this procedure. Though our protocol for broadcasting relies on phase sensitive and state-dependent cloners, we find that it is successful for various combinations of the squeezing parameter and the phase of the amplifiers.

We conclude by noting some possible off-shoots of our present study. It may be interesting to investigate other cloning protocols for continuous variables[9, 11] in order to see if they could lead to more efficient broadcasting. Whether broadcasting is possible for mixed states in general[5], is itself an open question for infinite dimensional systems. Furthermore, the possibility of generating three-mode quantum channels through local operations[14] useful for communications could be explored with photons. Finally, it may be noted that the first experimental demonstration of continuous variable cloning has been reported recently[15], and with further development it could be feasible to experimentally broadcast entangled states of continuous variables too.

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